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SIMPLE HINTS ON PLOTTING GRAPHS IN ANALYTIC GEOMETRY.

By AUBREY KEMPNER, Urbana, Ill.

In courses in analytic geometry, much importance is generally attached to the problem of plotting curves from given equations. It is therefore surprising that in most textbooks on analytic geometry some very simple and effective hints on plotting should be omitted. The contents of the present note are well known; for §§ III, IV compare the article in the MONTHLY for November, 1916, on "Graphical constructions for a function of a function and for a function given by a pair of parametric equations," by Professor W. H. Roever and his reference to Professor E. H. Moore.

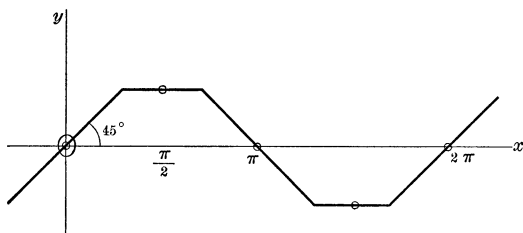


FIG. 1.

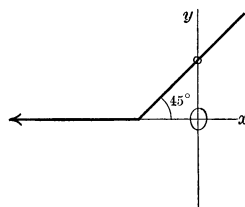


FIG. 3.

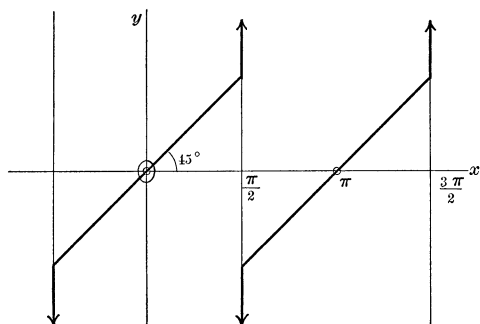


FIG. 2.

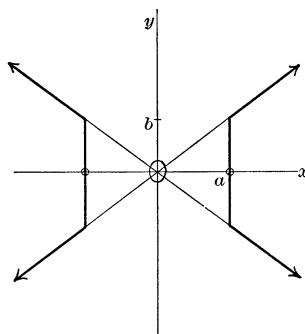


FIG. 4a.

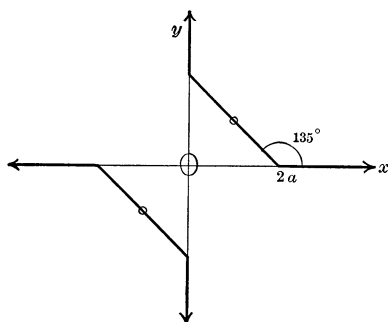


FIG. 4b.

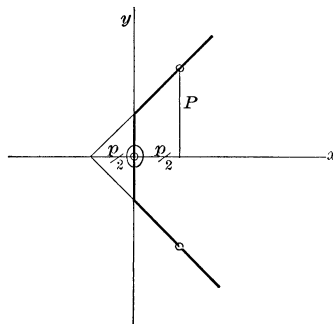


FIG. 5.

I. In many cases it is very easy to construct a few tangents to the curve with the corresponding points of tangency, and thus to gain a skeleton of the curve. Besides being a help in the actual plotting, such a skeleton is a valuable aid to the memory. In Figs. 1-5 these skeletons are constructed for certain standard curves. The unit of measurement is supposed to be the same on both axes. Asymptotes are marked with an arrow-tip.

Figure 1: $y = \sin x$ (x in radians¹), and similarly $y = \cos x$, and $y = \arcsin x$, $y = \arccos x$.

Figure 2: $y = \tan x$, and similarly $y = \cot x$, $y = \arctan x$, and $y = \text{arccot } x$.

Figure 3: $y = e^x$, and similarly $y = \log_e x$.

Figure 4a: The well-known figure for the hyperbola $x^2/a^2 - y^2/b^2 = 1$. (The corresponding figure for the ellipse consists simply of a rectangle, and is not here drawn.)

Figure 4b: The equilateral hyperbola $x \cdot y = a^2$.

Figure 5: The parabola $y^2 = 2px$.

II. To plot the graph of an equation of the type $y^2 = f(x)$ (or, similarly, of $x^2 = \varphi(y)$).

Rule: Construct the curve whose equation is $Y = f(x)$, using for Y -axis the original y -axis, and change the ordinates,² replacing Y by $y = \sqrt{Y}$.

The following relations between the curves $y^2 = f(x)$ and $Y = f(x)$ are evident:

(a) $y^2 = f(x)$ is symmetric with respect to x -axis.

(b) Corresponding to every loop of $Y = f(x)$ above the x -axis, $y^2 = f(x)$ has an oval.

(c) The parts of $Y = f(x)$ below the x -axis do not yield real points of $y^2 = f(x)$.

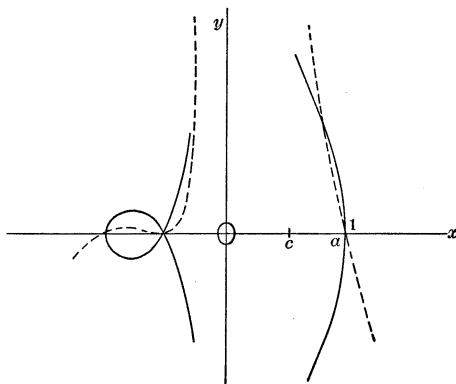


FIG. 6.

¹ The fraction $\frac{22}{7}$, as an approximate value for π , is exact to about $\frac{1}{2000}$ of the value of π , and is therefore sufficiently accurate for all graphical purposes.

² This method of construction is really nothing but a particularly simple case of changing, along the Y -axis, from a uniform scale to a certain non-uniform scale. See, for example, C. RUNGE, *Graphical Methods*, 1912, § 6, for the general method.

(d) When $0 < Y < 1$, the curve $y^2 = f(x)$ lies above the curve $Y = f(x)$; and when $Y > 1$ it lies below.

(e) The curves $Y = f(x)$, $y^2 = f(x)$ cross each other in all points where $Y = 0$ and where $Y = 1$.

Curves of this type are, for example, the conic sections, the semi-cubical parabola, the Cissoid, the Strophoid, the Conchoid, the Versiera, when the equations are given in the forms commonly used.

Figure 6: The conchoid $y^2 = \frac{1}{x^2} \cdot (x + c)^2 \cdot (a^2 - x^2)$, where $a = 1$, $c = \frac{1}{2}$. The

dotted curve is $Y = \frac{1}{x^2} \cdot (x + c)^2 \cdot (a^2 - x^2)$.

Figure 7: The parabola $y^2 = 2px$ (dotted line $Y = 2px$).

Figure 8: The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, or $y^2 = b^2 - \frac{b^2}{a^2}x^2$ (dotted curve $Y = b^2 - \frac{b^2}{a^2}x^2$),

and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, or $y^2 = -b^2 + \frac{b^2}{a^2}x^2$ (dotted curve $Y = -b^2 + \frac{b^2}{a^2}x^2$). The left half of the figure refers to the ellipse, the right half to the hyperbola. The two parabolas are congruent. $b = \frac{1}{2}\sqrt{2}$, $a = 1$.

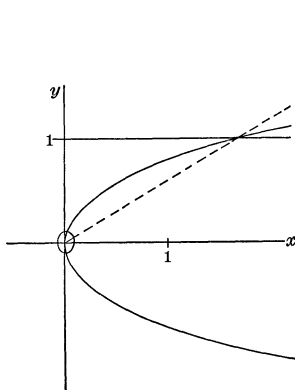


FIG. 7.

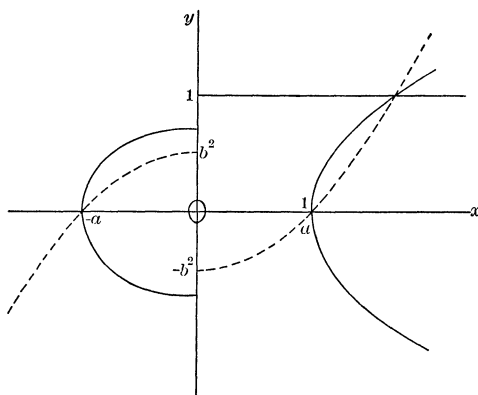


FIG. 8.

III. To construct the graph of $F(x, y) = 0$ when the equation is given in parametric form: $x = \varphi(t)$, $y = \psi(t)$.

*Rule:*¹ In a system of coördinates I, II (see Fig. 9) plot first $y = \psi(t)$, using I for the t -axis and II for the y -axis, and also $x = \varphi(t)$, using II for the t -axis and I for the x -axis. Next, draw the line of slope unity through the origin, assume any point on this line, draw through this point a line parallel to the axis II to its intersection (D in Fig. 9) with the curve $y = \psi(t)$, another line parallel to I to its intersection (E in Fig. 9) with the curve $x = \varphi(t)$, and complete the rec-

¹ Compare Professor Roever's article referred to above.

tangle. The fourth vertex (G in Fig. 9) is a point of $F(x, y) = 0$, with I for x -axis and II for y -axis.

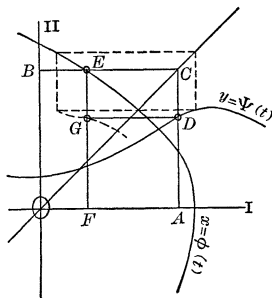


FIG. 9.

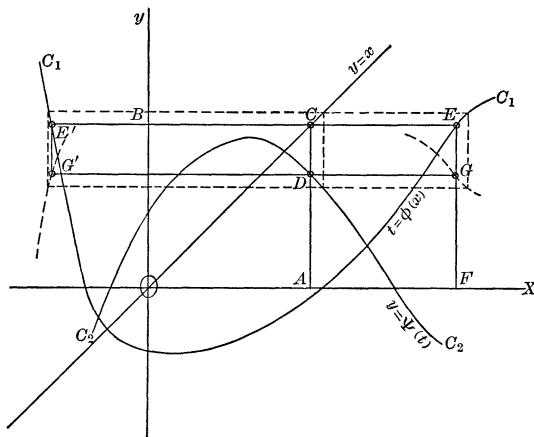


FIG. 10.

Proof: $AD = \psi(OA)$, $BE = \varphi(OB)$. (See Fig. 9.) Let $OA = OB = t$, then $BE = OF = \varphi(t) = x$, and $AD = FG = \psi(t) = y$.

IV. By interchanging x and t in $x = \varphi(t)$ we obtain

$$t = \varphi(x), \quad y = \psi(t) = \psi(\varphi(x))$$

and have thus the following rule for plotting the graph of a function of a function.

Rule: Plot in the $x - y$ system of coördinates the two curves (see Fig. 10)

C_1 : $t = \varphi(x)$, using for t -axis the y -axis,

C_2 : $y = \psi(t)$, using for t -axis the x -axis.

Draw the line $y = x$, assume any point C on it, draw through this point a line parallel to the y -axis to its intersection (D) with C_2 , and another line parallel to the x -axis to its intersections (E and E' in Fig. 10) with C_1 , and complete the rectangle. The fourth vertices (G and G' in Fig. 10) are points of the graph of $y = \psi(\varphi(x))$.¹

Proof: $AD = \psi(OA)$, $OB = \varphi(BE)$. (See Fig. 10.) Let $OA = OB = t$, then $AD = FG = \psi(t)$, $t = \varphi(BE) = \varphi(OF)$, and $FG = \psi(\varphi(OF))$.

In order to reduce the number of cuts, no illustrative examples for Sections III, IV have been given. The reader is referred to the examples worked out in Professor Roever's paper.

From the rule of construction in the present paragraph one can derive in geometrical form the conditions which must be satisfied in order to have $\psi(\varphi(x)) \equiv \varphi(\psi(x))$.

By taking for φ and ψ the same function one finds a simple method for

¹ For another kind of graphical representation of a function of a function compare, for example, C. RUNGE, *Graphical Methods*, 1912, § 6.

plotting the iterated function $\phi_2(x) = \phi(\phi(x))$; repeating the process for $\phi_2(x)$ instead of for $\phi(x)$, $y = \phi_4(x) = \phi\phi\phi\phi(x)$, and in the same way $y = \phi_k(x)$, may be plotted, when k is any power of 2. When k is not a power of 2, the construction of this paragraph generally leads to complicated figures.

A COURSE IN GEOMETRY FOR COLLEGE JUNIORS AND SENIORS.¹

By J. N. VAN DER VRIES, University of Kansas.

Courses in geometry for college juniors and seniors are usually given under the title "Modern Geometry." In many instances, the scope of the course is restricted by designating it as a course either in "Modern Synthetic Geometry" or in "Modern Analytic Geometry." An examination of the catalogs of a number of the leading American universities and colleges shows that there is no cleancut agreement as to the content of a course under any of these three titles. The term "Modern Geometry" seems to have grown up in our mathematical nomenclature for the purpose of distinguishing a college course in geometry under this name from the more elementary course of the secondary school, this latter course being the *ancient* geometry of the Greeks practically unchanged. This ancient geometry was, as Professor Cajori expresses it in his history, decidedly special. In fact, one of its principal characteristics was a complete lack of general principles and methods. Courses in modern geometry do not differ from the courses in elementary geometry so much in their content as in the methods by which they produce not only the well-known results of ancient geometry but also many results which could not be obtained by the equipment and methods of ancient geometry.

The college junior and senior ready to begin a course in modern geometry has had the usual amount of this elementary algebra and geometry, the latter for the larger part, if not entirely, the geometry of the plane. He will also have had the usual amount of college algebra, trigonometry, analytic geometry and calculus with some applications to mechanics and geometry. He may in some instances have had courses in unified mathematics both in his preparatory work and in his freshman year in college, but the various subjects will in most cases have been taught within well-defined boundary lines with no attempt at generalization or coördination. As a rule nowhere in his course has the student had much work tending to emphasize the homogeneity of all his work in *geometry* and the general principles which underlie it in its entirety. It is the belief of the writer that the chief aim of the course for college juniors and seniors should be to remedy the defects of the elementary geometry mentioned above by supplying the generalizing methods and principles towards which modern thought has contributed so greatly. A brief historical review of the introduction of these new concepts will not be out of place here.

¹ Read before the Kansas Section of the Mathematical Association of America, March 18, 1916.